محاضرة ٤

أحمد [illegible]

16

Pressure vessels

1) Open end نهايات مفتوحة

a) thin wall b) thick well

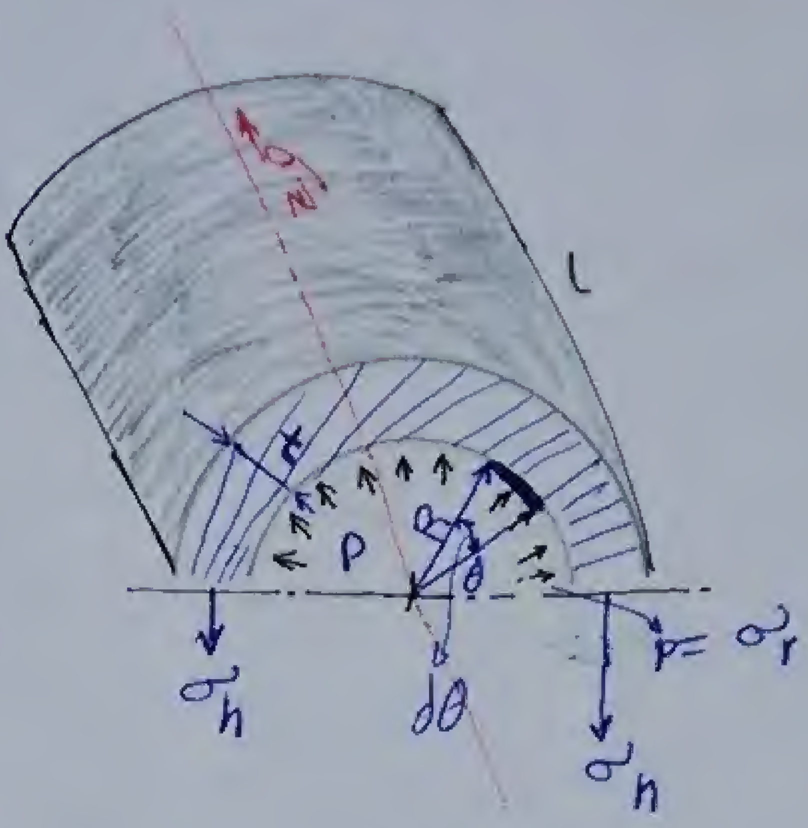
2) closed end Tanks

a) For thin wall pipes :-

$$\frac{t}{D} < 0.1$$

σ_h = hoop stress

σ_r = radial stress



$$\Sigma Force = 0$$

$$2\sigma_h \cdot t \cdot L = \int_0^{\pi} P \cdot R\, d\theta \cdot L \cdot \sin\theta$$

$$\sigma_h = \frac{PR}{2t}\left[-\cos\theta\right]_0^{\pi}$$

$$\sigma_h = \frac{PR}{2t}$$

$$\boxed{\sigma_r = P} \quad , \quad \boxed{\sigma_h = \frac{PR}{2t}}$$

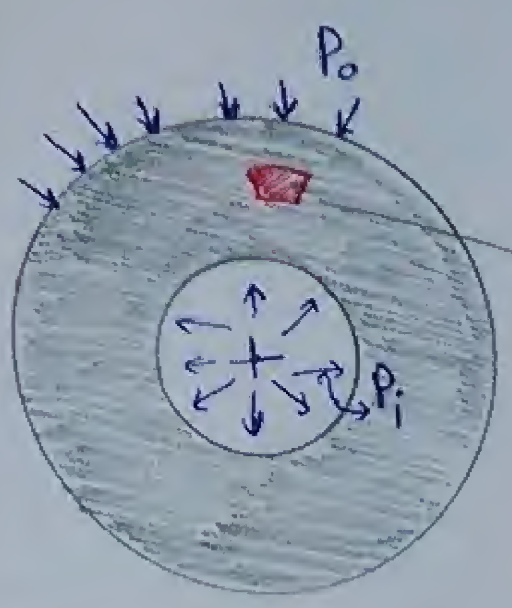
For open end $\sigma_z = 0$

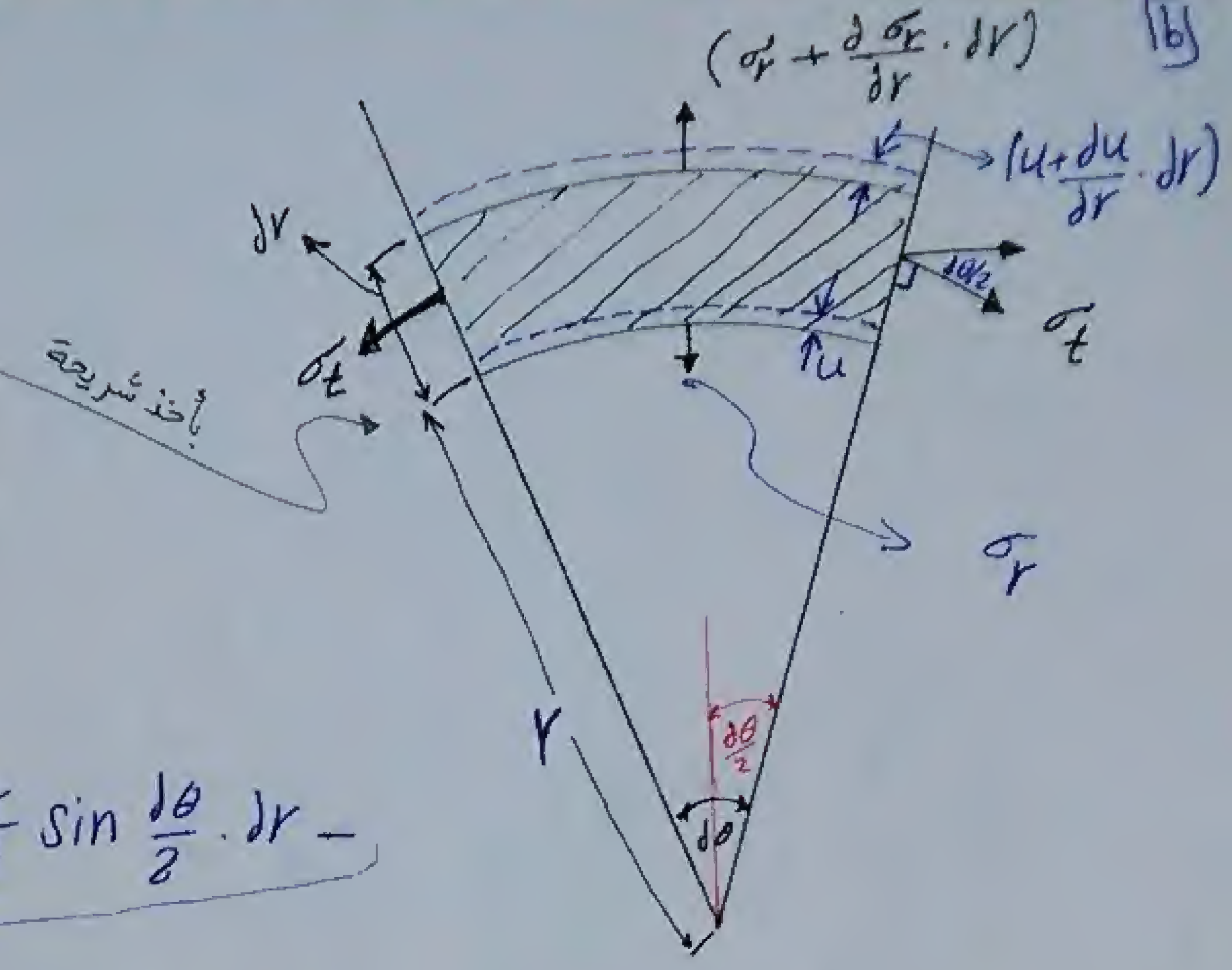
For colsed end

$$2\pi R t \cdot \sigma_z = P \cdot \frac{\pi D^2}{4}$$

$$\boxed{\sigma_z = \frac{PD}{4t} = \frac{\sigma_h}{2}}$$

b- for thick wall





$$\sigma_r \cdot r\,d\theta \cdot \cancel{1} + 2\,\frac{\sigma}{t} \sin\frac{d\theta}{2} \cdot dr -$$

لمساحة الشريحة

$$\left(\sigma_r + \frac{\partial \sigma_r}{\partial r} \cdot dr\right)(r + dr) \cdot d\theta$$

$$\sigma_r \cdot r\,d\theta + \sigma_t \cdot dr \cdot d\theta - \sigma_r \cdot r \cdot d\theta - \sigma_r \cdot dr\,d\theta - r\,\frac{\partial \sigma_r}{\partial r} \cdot dr \cdot d\theta - \frac{\partial \sigma_r}{\partial r} \cdot (dr)^2 \cdot d\theta = 0$$

$$\sigma_t - \sigma_r - r\,\frac{d\sigma_r}{dr} = 0 \longrightarrow (1)$$

$$\varepsilon_r = \frac{\partial u}{\partial r} \longrightarrow (2)$$

$$\varepsilon_t = \frac{u}{r} \longrightarrow (3)$$

$(\varepsilon_r + \varepsilon_t)$

$$\sigma_r = \frac{E}{(1-n)}\left(\frac{du}{dr} + n\,\frac{u}{r}\right) \longrightarrow (4)$$

$$\sigma_t = \frac{E}{1-n^2}\left(\frac{u}{r} + n\,\frac{du}{dr}\right) \longrightarrow (5)$$

بالتعويض من 4، 5 في (1)

$$\frac{d^2u}{dr^2} + \frac{1}{r} \cdot \frac{du}{dr} - \frac{u}{r^2} = 0 \longrightarrow (6)$$

$$u = c_1 r + \frac{c_2}{r} \longrightarrow (7)$$

$$\sigma_r = A - B/r^2 \longrightarrow (8)$$

$$\sigma_t = A + B/r^2 \longrightarrow (9)$$

Stresses Produced by internal or external Pressures.

Case (a) Internal Pressure only.

at $r=a$ $\quad \sigma_r = -P_i$

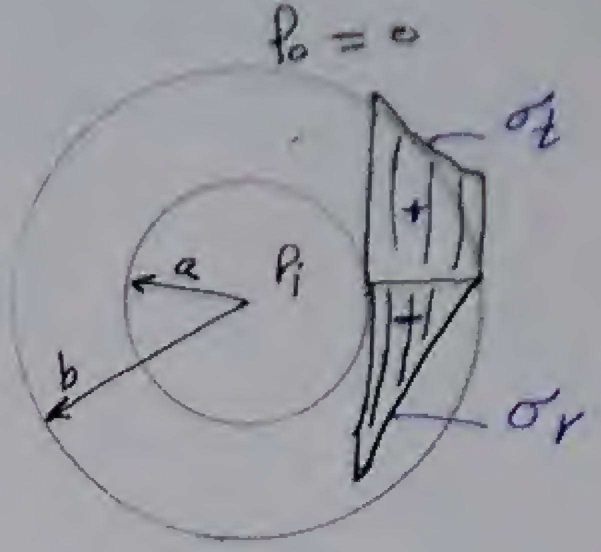
$r=b$ $\quad \sigma_r = 0$

$$A = P_i \frac{a^2}{(b^2-a^2)}$$

$$B = P_i \frac{a^2 b^2}{(b^2-a^2)}$$

$$\sigma_r = \frac{P_i a^2}{b^2-a^2}\left(1-\frac{b^2}{r^2}\right) \rightarrow (10)$$

$$\sigma_t = \frac{P_i a^2}{b^2-a^2}\left(1+\frac{b^2}{r^2}\right) \rightarrow (11)$$



Case (b) External Pressure only

$\sigma_r = 0$ at $r=a$

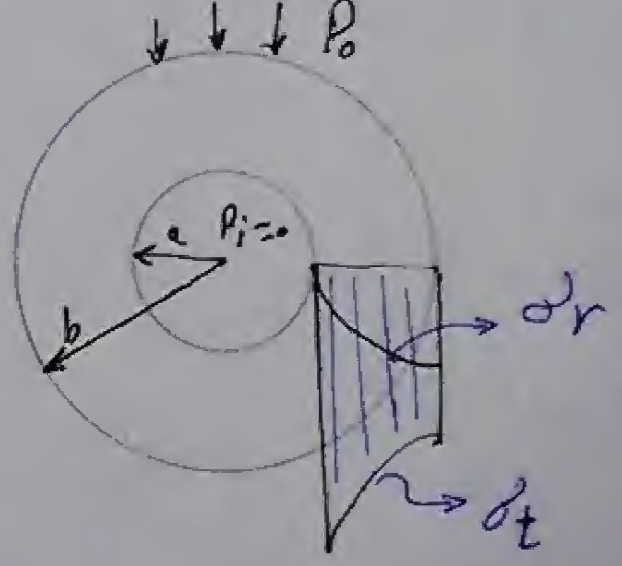
$\sigma_r = -P_o$ at $r=b$

$$A = P_o \frac{b^2}{b^2-a^2}$$

$$B = P_o \frac{a^2 b^2}{b^2-a^2}$$

$$\sigma_r = -\frac{P_o b^2}{(b^2-a^2)}\left(1-\frac{a^2}{r^2}\right) \rightarrow (12)$$

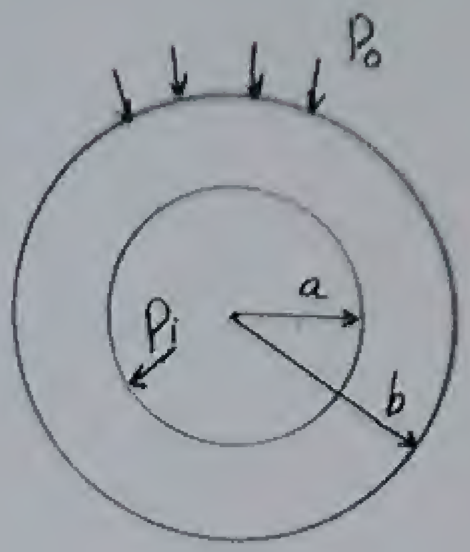
$$\sigma_t = -\frac{P_o b^2}{(b^2-a^2)}\left(1+\frac{a^2}{r^2}\right) \rightarrow (13)$$



Case 3 Internal Pressure & eXterant Pressure.

$\sigma_r = -P_i \rightarrow$ at $r = a$

$\sigma_r = -P_o \rightarrow$ at $r = b$



$$\sigma_r = \frac{a^2 \cdot P_i - b^2 \cdot P_o}{(b^2 - a^2)} - \frac{(P_i - P_o)a^2 b^2}{r^2(b^2 - a^2)}$$

$$\sigma_t = \frac{a^2 P_i - b^2 P_o}{b^2 - a^2} + \frac{(P_i - P_o)a^2 b^2}{r^2(b^2 - a^2)}$$

Deformation of the cylinders:

$$\varepsilon_r = \frac{1}{E}(\sigma_r - n\sigma_t) \rightarrow = \frac{1}{E}\left[A - \frac{B}{r^2} - nA - n\frac{B}{r^2}\right] = \frac{1}{E}\left[A(1-n) - \frac{B}{r^2}(1+n)\right] \rightarrow (16)$$

$$\varepsilon_t = \frac{1}{E}(\sigma_t - n\sigma_r)$$

$$\varepsilon_t = \frac{1}{E}\left[A(1-n) + \frac{B}{r^2}(1+n)\right] \rightarrow (17)$$

$\because \quad \varepsilon_t = \frac{u}{r} \qquad u = r \cdot \varepsilon_t$

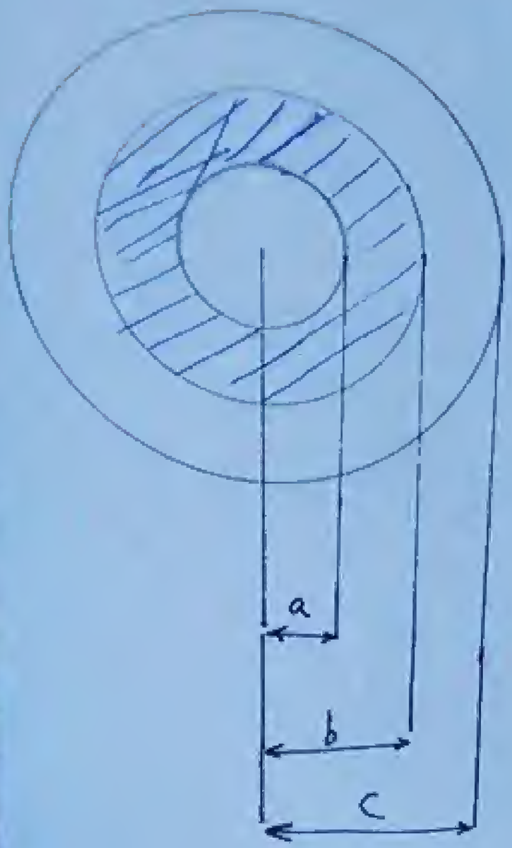
$$\therefore u = \frac{1}{E}\left[A(1-n)r + \frac{B}{r}(1+n)\right] \rightarrow (18)$$

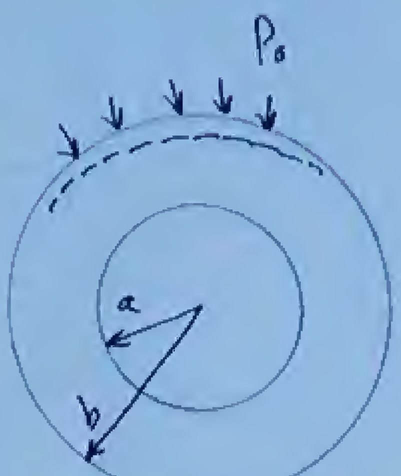
$$\varepsilon_z = \frac{1}{E}(\sigma_z - n\sigma_r - n\sigma_t)$$

For Long pipe $\varepsilon_z = 0$

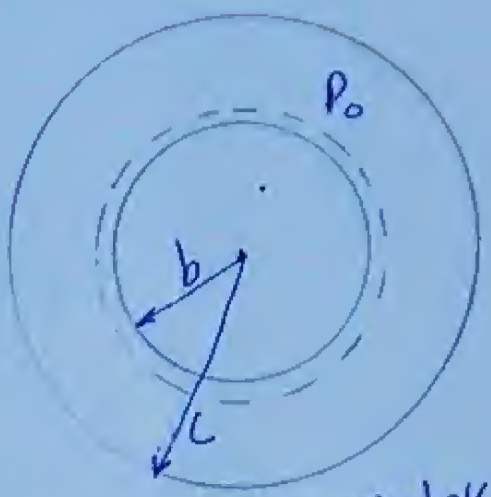
short pipe $\sigma_z = 0$

Pressures due to shrink Fits





inner cylinder



outer cylinder

$$P = \frac{E \cdot \delta}{b} \, \frac{(c^2-b^2)(b^2-a^2)}{2b^2(c^2-a^2)} \longrightarrow (19)$$

$$(\sigma_r)_{r=b} = -P \longrightarrow (20)$$

$$(\sigma_t)_{r=b} = + \frac{P(b^2+c^2)}{c^2-b^2} \longrightarrow (21)$$

$$\tau_{max} = \frac{1}{2}(\sigma_t - \sigma_r) = \frac{Pc^2}{c^2-b^2}.$$